A computer based classification of caps in PG(5,2)

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Abstract

In this paper we present the complete classification of caps in PG(5,2). These results have been obtained using a computer based exhaustive search that exploits projective equivalence.

1 Introduction

In the projective space PG(r,q) over the Galois Field GF(q), a n-cap is a set of n points no 3 of which are collinear. A n-cap is called complete if it is not contained in a (n+1)-cap. For a detailed description of the most important properties of these geometric structures, we refer the reader to [4]. In the last decades the problem of determining the spectrum of the sizes of complete caps has been the subject of a lot of researches. For a survey see [1]. In this work we search for the classification of complete and incomplete caps in PG(5,2), using an exhaustive search algorithm. In Section 2 the algorithm utilized is illustrated; in Section 3 the complete list of non equivalent complete and incomplete caps is presented.

2 The searching algorithm

In this section the algorithm utilized is presented. Our goal is to obtain the classification of complete and incomplete caps in PG(5,2). It is not restrictive to suppose that a cap in PG(5,2) contains this six points:

Then we define the set Cand of all the points lying no 2-secant of \mathcal{R} . We introduce in Cand the following equivalence relationship:

$$P \sim Q \iff \mathcal{C} \cup \{P\} \cong \mathcal{C} \cup \{Q\},$$

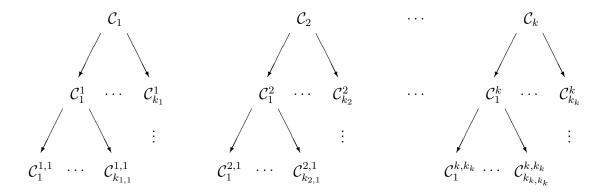
where \cong means that the two sets are projectively equivalent. This relationship spreads the candidates in equivalent classes C_1, \ldots, C_k .

The choice of the next point to add to the building cap can be made only among the representatives of the equivalent classes, in fact two caps one containing $\mathcal{C} \cup \{P\}$ and the other one $\mathcal{C} \cup \{Q\}$, with P and Q in $\mathcal{C}_{\overline{i}}$, are equivalent by definition of orbit.

Suppose now that we have construct all the caps containing $C \cup \{P_i\}$, with $i \leq \overline{i}$. Considering the caps containing $C \cup \{P_j\}$ with $\overline{i} < j$, all the points of the classes C_k with $k < \overline{i}$ can be avoided. In fact a cap containing $C \cup \{P_j\} \cup \{\overline{P}_k\}$, with $\overline{P}_k \in C_k$ and $k < \overline{i}$, is projectively equivalent to a cap containing $C \cup \{P_k\} \cup \{P_j\}$, already studied.

When we add a new point to the cap, we can divide all the remaining candidates in equivalence classes, as above. Two points P and Q are in relationship with the j-th class C_j , i.e. $P \sim_j Q$, if $C \cup \{P_j\} \cup \{P\}$ and $C \cup \{P_j\} \cup \{Q\}$ are projectively equivalent.

At the *m*-th step of the extension process if the cap $\mathcal{C} \cup \{P_{i_m}\} \cup \ldots \cup \{P_{i_m}^{i_1\ldots i_{m-1}}\} \cup \{P\}$ is projectively equivalent to the cap $\mathcal{C} \cup \{P_{i_m}\} \cup \ldots \cup \{P_{i_m}^{i_1\ldots i_{m-1}}\} \cup \{Q\}$ with $P_s^{i_1\ldots i_r} \in \mathcal{C}_s^{i_1\ldots i_r}$, then P and Q are in relationship $(P \sim_{i_1\ldots i_m} Q)$ and they belong to the same class $\mathcal{C}_{m+1}^{i_1\ldots i_m}$. Iterating the process we can build a tree similar to the following:



The tree is important to restrict the number of candidates in the extension process. Suppose that we have generated a n-cap containing the cap $\mathcal{C} \cup \{P_{i_1}\} \cup \{P_{i_2}^{i_1}\} \cup \ldots \cup \{P_{i_m}^{i_1\ldots i_{m-1}}\} \cup \{P\}$, after having generated n-caps containing $\mathcal{C} \cup \{P_j\}$ with $j < i_1, \ \mathcal{C} \cup \{P_{i_1}\} \cup \{P_j^{i_1}\}$ with $j < i_2, \ldots, \ \mathcal{C} \cup \{P_{i_1}\} \cup \{P_{i_2}^{i_1}\} \cup \ldots \cup \{P_j^{i_1\ldots i_{m-1}}\}$ with $j < i_m$, with $p_s^{i_1\ldots i_r} \in \mathcal{C}_s^{i_1\ldots i_r}$. Then the

points belonging to $C_1 \cup \ldots \cup C_{i_1-1} \cup C_1^{i_1} \cup \ldots \cup C_{i_2-1}^{i_1} \cup \ldots \cup C_1^{i_1 \dots i_{m-1}} \cup \ldots \cup C_{i_m-1}^{i_1 \dots i_{m-1}}$ can be avoided, because a cap containing one of them is equivalent to one already found. For example a n-cap containing $C \cup \{P_{i_1}\} \cup \ldots \cup \{P_{i_m}^{i_1 \dots i_{m-1}}\} \cup \{P\} \cup \{Q\}$ with $Q \in C_h$ for some $h < i_1$ is equivalent to a n-cap containing $C \cup \{P_h\}$, which is already found.

3 Results

In this Section all non equivalent caps, complete and incomplete, in PG(5,2) are presented.

3.1 Non-equivalent caps K in PG(5,2)

This table shows the number and the type of the non equivalent examples of all the caps.

$ \mathcal{K} $	# COMPLETE	# INCOMPLETE	$ \mathcal{K} $	# COMPLETE	# INCOMPLETE
	CAPS	CAPS		CAPS	CAPS
7	0	4	20	1	23
8	0	7	21	0	16
9	0	12	22	0	15
10	0	24	23	0	9
11	0	34	24	0	8
12	0	43	25	0	5
13	1	46	26	0	4
14	0	49	27	0	2
15	0	44	28	0	2
16	0	48	29	0	1
17	5	35	30	0	1
18	1	32	31	0	1
19	0	25	32	1	0

Table 1: Number and type of non equivalent examples

3.2 Description of the caps

In this section we describe each cap of size k+1, with $7 \ge k \ge 31$, as union of a cap of size k and a point in PG(5,2). We start from these non equivalent examples of 7-incomplete caps.

		С	ΑP	1					C	AP	2		
1	1	0	0	0	0	0	1	1	0	0	0	0	0
0	1	1	0	0	0	0	0	0	1	0	0	0	0
0	1	0	1	0	0	0	0	0	0	1	0	0	0
0	1	0	0	1	0	0	0	0	0	0	1	0	0
0	1	0	0	0	1	0	1	0	0	0	0	1	0
0	1	0	0	0	0	1	1	0	0	0	0	0	1
		С	ΑP	3					C	AP	4		
1	0	C 0	AP 0	3	0	1	1	0	0 0	AP 0	4	0	1
1 0	0				0 0	1 1	$\begin{matrix} 1 \\ 0 \end{matrix}$					0 0	1 1
_	-	0	0	0				1	0	0	0	-	
0	1	0	0	0	0	1	0	1 0	0	0	0	0	1
0	1 0	0 0 1	0 0 0	0 0 0	0	1 0	0	1 0 0	0 0 1	0 0 0	0 0 0	0	1 1

We will call these caps $C_7^1, C_7^2, C_7^3, C_7^4$ respectively. They have stabilizer (\mathcal{O}) of size

Table 2: Cap \mathcal{K} with $|\mathcal{K}| = 7$

\mathcal{K}	STABILIZER
\mathcal{C}_7^1	$ \mathcal{O} = 5040$
\mathcal{C}_7^2	$ \mathcal{O} = 144$
\mathcal{C}_7^3	$ \mathcal{O} = 240$
\mathcal{C}_7^4	$ \mathcal{O} = 720$

In the following tables the symbol C_i^j indicates the *j*-th cap of size i and \overline{C}_i^j means that the cap is complete. If it is possible to write a cap in two different ways, we choose that one with the lowest value of j.

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
	$\mathcal{C}_7^2 \cup \{(1,1,0,1,1,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^2_8	$C_7^3 \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 144$
\mathcal{C}_8^3	$C_7^2 \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 96$		$C_7^2 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 192$
\mathcal{C}_8^5	$C_7^2 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 1152$	\mathcal{C}_8^6	$C_7^2 \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 72$
\mathcal{C}^7_8	$C_7^1 \cup \{(1,1,0,1,1,0)\}$	$ \mathcal{O} = 144$			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}_9^1	$C_8^1 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}_9^2	$C_8^1 \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}_9^3	$C_8^3 \cup \{(1,1,0,0,1,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}_9^4	$C_8^4 \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}_9^5	$C_8^3 \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}_9^6	$C_8^3 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 384$
\mathcal{C}^7_9	$C_8^1 \cup \{(1, 1, 1, 0, 1, 0)\}$	$ \mathcal{O} = 128$	\mathcal{C}_9^8	$C_8^1 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 288$
\mathcal{C}_9^9	$C_8^1 \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 32$	\mathcal{C}_9^{10}	$C_8^3 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 336$
\mathcal{C}_9^{11}	$C_8^3 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 144$	\mathcal{C}_9^{12}	$C_8^1 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{10}	$C_9^2 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 336$	\mathcal{C}^2_{10}	$C_9^3 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 192$
\mathcal{C}^3_{10}	$C_9^2 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^4_{10}	$C_9^3 \cup \{(0,1,1,1,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^5_{10}	$C_9^2 \cup \{(1,0,1,1,1,0)\}$	$ \mathcal{O} = 192$	\mathcal{C}^6_{10}	$C_9^1 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^7_{10}	$C_9^6 \cup \{(1, 1, 0, 0, 0, 1)\}$	$ \mathcal{O} = 3840$	\mathcal{C}^8_{10}	$C_9^3 \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 72$
\mathcal{C}^9_{10}	$C_9^{10} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 2688$	\mathcal{C}^{10}_{10}	$C_9^7 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^{11}_{10}	$C_9^1 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 144$	\mathcal{C}^{12}_{10}	$C_9^3 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{13}_{10}	$C_9^3 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	C_{10}^{14}	$C_9^4 \cup \{(1,1,0,1,0,1)\}$	\mathcal{D}_6
\mathcal{C}^{15}_{10}	$C_9^4 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 120$	\mathcal{C}_{10}^{16}	$C_9^6 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 384$
\mathcal{C}^{17}_{10}	$C_9^2 \cup \{(0,0,1,1,1,1)\}$	$ \mathcal{O} = 192$	\mathcal{C}^{18}_{10}	$\mathcal{C}_9^{10} \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 1008$
\mathcal{C}_{10}^{19}	$C_9^1 \cup \{(1,0,0,1,0,1)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	C_{10}^{20}	$C_9^1 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{21}_{10}	$C_9^1 \cup \{(1,1,1,0,1,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	\mathcal{C}^{22}_{10}	$C_9^2 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 192$
C_{10}^{23}	$C_9^3 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 64$	C_{10}^{24}	$C_9^5 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 144$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{11}	$C_{10}^6 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 144$	\mathcal{C}^2_{11}	$C_{10}^6 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$
\mathcal{C}^3_{11}	$C_{10}^9 \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 1152$	\mathcal{C}^4_{11}	$C_{10}^4 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 32$
\mathcal{C}^5_{11}	$C_{10}^2 \cup \{(0,1,1,1,0,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^6_{11}	$C_{10}^3 \cup \{(1,1,1,0,0,1)\}$	$ \mathcal{O} = 96$
\mathcal{C}^7_{11}	$C_{10}^9 \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 1008$	\mathcal{C}^8_{11}	$C_{10}^{10} \cup \{(0,0,1,1,1,0)\}$	$\mathcal{D}_4 imes\mathcal{C}_2$
\mathcal{C}^9_{11}	$C_{10}^9 \cup \{(1,1,0,0,1,0)\}$	$ \mathcal{O} = 1384$	\mathcal{C}^{10}_{11}	$C_{10}^{14} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 120$
C_{11}^{11}	$C_{10}^3 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 384$	\mathcal{C}^{12}_{11}	$\mathcal{C}_{10}^5 \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 32$
C_{11}^{13}	$C_{10}^8 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 48$	C_{11}^{14}	$\mathcal{C}_{10}^3 \cup \{(1,1,1,0,1,1)\}$	\mathcal{D}_4
\mathcal{C}^{15}_{11}	$C_{10}^1 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 720$	C_{11}^{16}	$C_{10}^8 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{17}_{11}	$C_{10}^4 \cup \{(1,0,0,1,1,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	C_{11}^{18}	$\mathcal{C}_{10}^4 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 48$
C_{11}^{19}	$C_{10}^2 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 32$	C_{11}^{20}	$C_{10}^{22} \cup \{(0,1,0,1,1,1)\}$	$ \mathcal{O} = 1920$
\mathcal{C}^{21}_{11}	$C_{10}^6 \cup \{(1,1,1,0,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	\mathcal{C}^{22}_{11}	$C_{10}^8 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$
C_{11}^{23}	$\mathcal{C}_{10}^{19} \cup \{(1,1,0,0,0,1)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	C_{11}^{24}	$\mathcal{C}_{10}^6 \cup \{(1,0,0,1,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$
C_{11}^{25}	$C_{10}^6 \cup \{(0,1,1,1,0,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	C_{11}^{26}	$C_{10}^1 \cup \{(1,0,1,1,1,0)\}$	$ \mathcal{O} = 1334$
\mathcal{C}^{27}_{11}	$C_{10}^2 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	C_{11}^{28}	$\mathcal{C}_{10}^1 \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 48$
C_{11}^{29}	$C_{10}^7 \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 192$	C_{11}^{30}	$C_{10}^5 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 48$
C_{11}^{31}	$C_{10}^7 \cup \{(1,1,0,1,1,0)\}$	$ \mathcal{O} = 384$	C_{11}^{32}	$C_{10}^4 \cup \{(1,1,1,0,1,1)\}$	\mathcal{D}_5
C_{11}^{33}	$C_{10}^2 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 48$	C_{11}^{34}	$C_{10}^7 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 1920$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{12}	$C_{11}^6 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^2_{12}	$C_{11}^{13} \cup \{(1,1,1,0,1,1)\}$	\mathcal{D}_4
\mathcal{C}^3_{12}	$\mathcal{C}_{11}^2 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^4_{12}	$\mathcal{C}_{11}^{10} \cup \{(1,1,0,0,1,0)\}$	\mathcal{D}_5
\mathcal{C}^5_{12}	$\mathcal{C}_{11}^4 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 128$	\mathcal{C}^6_{12}	$\mathcal{C}_{11}^6 \cup \{(1,0,1,1,1,0)\}$	$ \mathcal{O} = 1152$
\mathcal{C}^7_{12}	$\mathcal{C}_{11}^{34} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 23040$	\mathcal{C}^8_{12}	$\mathcal{C}_{11}^5 \cup \{(1,1,1,1,1,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^9_{12}	$C_{11}^{27} \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_6	\mathcal{C}^{10}_{12}	$\mathcal{C}_{11}^4 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 384$
\mathcal{C}^{11}_{12}	$\mathcal{C}_{11}^1 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$	\mathcal{C}^{12}_{12}	$\mathcal{C}_{11}^7 \cup \{(1,1,0,0,1,0)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{13}_{12}	$\mathcal{C}_{11}^3 \cup \{(1,1,0,0,1,0)\}$	$ \mathcal{O} = 128$	\mathcal{C}^{14}_{12}	$\mathcal{C}_{11}^{13} \cup \{(1,1,1,0,0,0)\}$	$\mathcal{D}_4 imes\mathcal{C}_2 imes\mathcal{C}_2$
\mathcal{C}^{15}_{12}	$C_{11}^4 \cup \{(0,0,1,1,1,1)\}$	$ \mathcal{O} = 32$	\mathcal{C}^{16}_{12}	$C_{11}^{21} \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{17}_{12}	$C_{11}^{17} \cup \{(1,1,1,0,1,1)\}$	\mathcal{D}_4	\mathcal{C}^{18}_{12}	$C_{11}^{14} \cup \{(0,0,0,1,1,1)\}$	\mathcal{D}_4
\mathcal{C}^{19}_{12}	$\mathcal{C}_{11}^5 \cup \{(0,0,0,1,1,1)\}$	\mathcal{D}_4	\mathcal{C}^{20}_{12}	$\mathcal{C}_{11}^{13} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{21}_{12}	$\mathcal{C}_{11}^{32} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 120$	\mathcal{C}^{22}_{12}	$\mathcal{C}_{11}^9 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 1536$
\mathcal{C}^{23}_{12}	$\mathcal{C}_{11}^{13} \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 120$	C_{12}^{24}	$\mathcal{C}_{11}^{25} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{25}_{12}	$\mathcal{C}_{11}^9 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 256$	\mathcal{C}^{26}_{12}	$C_{11}^{12} \cup \{(1,1,1,0,1,0)\}$	$ \mathcal{O} = 128$
\mathcal{C}^{27}_{12}	$C_{11}^1 \cup \{(1,0,0,1,0,1)\}$	\mathcal{D}_6	C_{12}^{28}	$\mathcal{C}_{11}^{16} \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{29}_{12}	$C_{11}^{20} \cup \{(0,1,1,0,1,1)\}$	$ \mathcal{O} = 192$	C_{12}^{30}	$\mathcal{C}_{11}^4 \cup \{(1,1,1,0,1,1)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$
C_{12}^{31}	$\mathcal{C}_{11}^{20} \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 3840$	C_{12}^{32}	$\mathcal{C}_{11}^4 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 96$
C_{12}^{33}	$C_{11}^6 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 96$	C_{12}^{34}	$C_{11}^2 \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_6
\mathcal{C}^{35}_{12}	$C_{11}^{8} \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	C_{12}^{36}	$\mathcal{C}_{11}^{13} \cup \{(0,0,1,1,1,0)\}$	$\mathcal{D}_4 imes\mathcal{C}_2 imes\mathcal{C}_2$
C_{12}^{37}	$C_{11}^{22} \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 144$	C_{12}^{38}	$C_{11}^8 \cup \{(0,0,0,1,1,1)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$
C_{12}^{39}	$C_{11}^4 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 64$	C_{12}^{40}	$\mathcal{C}_{11}^4 \cup \{(0,0,1,1,1,0)\}$	\mathcal{D}_4
C_{12}^{41}	$C_{11}^2 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 240$	\mathcal{C}^{42}_{12}	$\mathcal{C}_{11}^9 \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 192$
C_{12}^{43}	$C_{11}^2 \cup \{(1,0,0,1,0,1)\}$	\mathcal{D}_4			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
$\overline{\mathcal{C}}_{13}^1$	$C_{12}^6 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 1152$	\mathcal{C}^2_{13}	$C_{12}^2 \cup \{(1,0,1,0,0,1)\}$	\mathcal{D}_6
\mathcal{C}^3_{13}	$C_{12}^{12} \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^4_{13}	$C_{12}^{28} \cup \{(1,1,1,1,0,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^5_{13}	$C_{12}^2 \cup \{(1,1,1,0,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	\mathcal{C}^6_{13}	$C_{12}^{11} \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$
\mathcal{C}^7_{13}	$\mathcal{C}_{12}^{11} \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 96$	\mathcal{C}^8_{13}	$\mathcal{C}_{12}^{14} \cup \{(0,1,1,1,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^9_{13}	$C_{12}^1 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 32$	\mathcal{C}^{10}_{13}	$\mathcal{C}_{12}^2 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 48$
$C_{13}^{\bar{1}\bar{1}}$	$C_{12}^{23} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 192$	C_{13}^{12}	$\mathcal{C}_{12}^{27} \cup \{(0,1,1,1,0,1)\}$	\mathcal{D}_6
C_{13}^{13}	$C_{12}^3 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 384$	C_{13}^{14}	$\mathcal{C}_{12}^{15} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^{15}_{13}	$\mathcal{C}_{12}^{12} \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 192$	C_{13}^{16}	$\mathcal{C}_{12}^{10} \cup \{(0,0,1,1,1,1)\}$	$ \mathcal{O} = 384$
C_{13}^{17}	$\mathcal{C}_{12}^{29} \cup \{(0,1,1,1,0,1)\}$	$ \mathcal{O} = 48$	C_{13}^{18}	$C_{12}^2 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 96$
C_{13}^{19}	$C_{12}^{24} \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 48$	C_{13}^{20}	$C_{12}^9 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 48$
C_{13}^{21}	$C_{12}^{34} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 144$	C_{13}^{22}	$C_{12}^8 \cup \{(0,0,0,1,1,1)\}$	$\mathcal{D}_4 imes\mathcal{C}_2$
C_{13}^{23}	$C_{12}^{14} \cup \{(1,0,1,0,1,0)\}$	$\mathcal{D}_4 imes \mathcal{C}_2$	C_{13}^{24}	$\begin{array}{c} \mathcal{C}_{12}^7 \cup \{(1,1,1,0,1,1)\} \\ \mathcal{C}_{22}^{22} \cup \{(2,2,1,1,1,0)\} \end{array}$	$ \mathcal{O} = 1152$
C_{13}^{25}	$C_{12}^3 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 48$	C_{13}^{26}	$C_{12}^{22} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 128$
C_{13}^{27}	$C_{12}^{16} \cup \{(1,1,0,1,0,1)\}$	$ \mathcal{O} = 192$	C_{13}^{28}	$C_{12}^{27} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$
C_{13}^{29}	$C_{12}^5 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$	C_{13}^{30}	$C_{12}^4 \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_4
C_{13}^{31}	$C_{12}^{42} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 2304$	C_{13}^{32}	$C_{12}^8 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 48$
C_{13}^{33}	$C_{12}^1 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 32$	C_{13}^{34}	$C_{12}^2 \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{35}_{13} \mathcal{C}^{37}_{13}	$\begin{array}{c} C_{12}^{13} \cup \{(1, 1, 1, 0, 0, 1)\} \\ C_{12}^{5} \cup \{(0, 1, 0, 0, 1, 1)\} \end{array}$	$ \mathcal{O} = 256$	\mathcal{C}^{36}_{13} \mathcal{C}^{38}_{13}	$C_{12}^3 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}_{13}^{39}	$\begin{array}{c} \mathcal{C}_{12} \cup \{(0,1,0,0,1,1)\} \\ \mathcal{C}_{12}^5 \cup \{(1,1,1,0,1,1)\} \end{array}$	$ \mathcal{O} = 32$ \mathcal{D}_8	\mathcal{C}_{13}^{40}	$\begin{array}{c} \mathcal{C}_{12}^2 \cup \{(0, 1, 1, 0, 0, 1)\} \\ \mathcal{C}_{12}^2 \cup \{(0, 1, 0, 0, 1, 1)\} \end{array}$	$\mathcal{D}_4 \ \mathcal{D}_6 imes \mathcal{C}_2$
\mathcal{C}_{13}^{41}	$C_{12} \cup \{(1,1,1,0,1,1)\}$ $C_{12}^4 \cup \{(1,1,1,0,0,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	\mathcal{C}^{13}_{13} \mathcal{C}^{42}_{13}	$\begin{array}{c} \mathcal{C}_{12} \cup \{(0,1,0,0,1,1)\} \\ \mathcal{C}_{12}^{28} \cup \{(1,1,1,0,0,0)\} \end{array}$	$ \mathcal{O}_6 \times \mathcal{C}_2 $ $ \mathcal{O} = 36$
\mathcal{C}_{13}^{43}	$\begin{array}{c} \mathcal{C}_{12} \cup \{(1,1,1,0,0,0)\} \\ \mathcal{C}_{12}^{36} \cup \{(1,0,1,1,0,0)\} \end{array}$	$ \mathcal{O} = 144$	\mathcal{C}_{13}^{44}	$C_{12} \cup \{(1, 1, 1, 0, 0, 0)\}$ $C_{12}^{10} \cup \{(1, 1, 1, 0, 1, 1)\}$	$ \mathcal{O} = 30$ $ \mathcal{O} = 48$
\mathcal{C}_{13}^{13} \mathcal{C}_{13}^{45}	$C_{12} \cup \{(1,0,1,1,0,0)\}$ $C_{12}^9 \cup \{(0,0,0,1,1,1)\}$	\mathcal{S}_3	\mathcal{C}^{13}_{13}	$\begin{array}{c} \mathcal{C}_{12} \cup \{(1,1,1,0,1,1)\} \\ \mathcal{C}_{12}^{13} \cup \{(0,1,1,1,0,1)\} \end{array}$	$ \mathcal{O} = 48$ $ \mathcal{O} = 64$
\mathcal{C}_{13}^{13} \mathcal{C}_{13}^{47}	$\begin{array}{c} \mathcal{C}_{12} \cup \{(0,0,0,1,1,1)\} \\ \mathcal{C}_{12}^2 \cup \{(0,1,1,1,0,1)\} \end{array}$	\mathcal{S}_3	c_{13}	$C_{12} \cup \{(0, 1, 1, 1, 0, 1)\}$	04
c_{13}	[0,1,1,1,0,1]	\mathcal{O}_3			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{14}	$C_{13}^{16} \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^2_{14}	$C_{13}^3 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^3_{14}	$C_{13}^{22} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 128$	\mathcal{C}^4_{14}	$C_{13}^2 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^5_{14}	$C_{13}^7 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 96$	\mathcal{C}^6_{14}	$C_{13}^6 \cup \{(1,1,0,1,0,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^7_{14}	$C_{13}^5 \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	\mathcal{C}^8_{14}	$C_{13}^6 \cup \{(0,0,0,1,1,1)\}$	$ \mathcal{O} = 36$
\mathcal{C}^9_{14}	$C_{13}^4 \cup \{(0,0,1,1,1,1)\}$	$ \mathcal{O} = 192$	\mathcal{C}^{10}_{14}	$C_{13}^9 \cup \{(1,0,1,1,0,0)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$
\mathcal{C}^{11}_{14}	$C_{13}^3 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^{12}_{14}	$C_{13}^6 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{13}_{14}	$C_{13}^8 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 128$	\mathcal{C}^{14}_{14}	$\mathcal{C}_{13}^5 \cup \{(0,0,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^{15}_{14}	$C_{13}^2 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_6	\mathcal{C}^{16}_{14}	$\mathcal{C}_{13}^2 \cup \{(0,1,1,1,0,1)\}$	\mathcal{D}_6
\mathcal{C}^{17}_{14}	$C_{13}^{27} \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 1536$	\mathcal{C}^{18}_{14}	$\mathcal{C}_{13}^{12} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{19}_{14}	$\mathcal{C}_{13}^2 \cup \{(0,1,0,0,1,1)\}$	\mathcal{D}_6	\mathcal{C}^{20}_{14}	$\mathcal{C}_{13}^5 \cup \{(1,0,1,1,1,1)\}$	\mathcal{D}_4
\mathcal{C}^{21}_{14}	$C_{13}^6 \cup \{(1,0,0,1,0,1)\}$	\mathcal{D}_4	\mathcal{C}^{22}_{14}	$\mathcal{C}_{13}^{39} \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 96$
C_{14}^{23}	$C_{13}^{15} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 2304$	C_{14}^{24}	$\mathcal{C}_{13}^2 \cup \{(0,1,0,1,0,1)\}$	\mathcal{D}_6
\mathcal{C}^{25}_{14}	$C_{13}^{18} \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 64$	C_{14}^{26}	$\mathcal{C}_{13}^{12} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{27}_{14}	$\mathcal{C}_{13}^2 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	C_{14}^{28}	$\mathcal{C}_{13}^{11} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 64$
C_{14}^{29}	$C_{13}^{42} \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 96$	C_{14}^{30}	$\mathcal{C}_{13}^5 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 32$
C_{14}^{31}	$\mathcal{C}_{13}^7 \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 48$	C_{14}^{32}	$\mathcal{C}_{13}^2 \cup \{(1,1,1,0,0,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$
C_{14}^{33}	$\mathcal{C}_{13}^9 \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 192$	C_{14}^{34}	$C_{13}^{10} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 48$
C_{14}^{35}	$\mathcal{C}_{13}^4 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_6	C_{14}^{36}	$\mathcal{C}_{13}^3 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 192$
C_{14}^{37}	$\mathcal{C}_{13}^6 \cup \{(1,0,1,1,1,0)\}$	$ \mathcal{O} = 32$	C_{14}^{38}	$C_{13}^2 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 32$
C_{14}^{39}	$C_{13}^{26} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 1536$	C_{14}^{40}	$\mathcal{C}_{13}^{32} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 384$
C_{14}^{41}	$\mathcal{C}_{13}^5 \cup \{(0,1,0,0,1,1)\}$	\mathcal{D}_4	C_{14}^{42}	$\mathcal{C}_{13}^8 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 144$
C_{14}^{43}	$\mathcal{C}_{13}^7 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 1152$	C_{14}^{44}	$\mathcal{C}_{13}^3 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 64$
C_{14}^{45}	$C_{13}^{18} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 384$	C_{14}^{46}	$C_{13}^{10} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 384$
C_{14}^{47}	$\mathcal{C}_{13}^{11} \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 576$	C_{14}^{48}	$\mathcal{C}_{13}^{26} \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 384$
C_{14}^{49}	$\mathcal{C}_{13}^4 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 36$			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{15}	$C_{14}^{19} \cup \{(0,1,1,0,0,1)\}$	\mathcal{D}_4	\mathcal{C}^2_{15}	$C_{14}^4 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$
\mathcal{C}^3_{15}	$C_{14}^7 \cup \{(0,0,1,1,1,0)\}$	\mathcal{S}_3	\mathcal{C}^4_{15}	$\mathcal{C}_{14}^7 \cup \{(1,0,1,0,0,1)\}$	$\mathcal{D}_4 imes\mathcal{C}_2$
\mathcal{C}^5_{15}	$\mathcal{C}_{14}^3 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 64$	\mathcal{C}^6_{15}	$\mathcal{C}_{14}^2 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 128$
\mathcal{C}^7_{15}	$\mathcal{C}_{14}^2 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^8_{15}	$\mathcal{C}_{14}^1 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 32$
\mathcal{C}^9_{15}	$C_{14}^{19} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^{10}_{15}	$\mathcal{C}_{14}^{25} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 64$
\mathcal{C}^{11}_{15}	$C_{14}^8 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 96$	\mathcal{C}^{12}_{15}	$\mathcal{C}_{14}^5 \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 32$
C_{15}^{13}	$C_{14}^1 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 96$	\mathcal{C}^{14}_{15}	$\mathcal{C}_{14}^7 \cup \{(1,1,1,1,1,0)\}$	\mathcal{D}_6
\mathcal{C}^{15}_{15}	$C_{14}^{11} \cup \{(0,1,1,1,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	\mathcal{C}^{16}_{15}	$\mathcal{C}_{14}^{29} \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 720$
\mathcal{C}^{17}_{15}	$C_{14}^{28} \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 384$	C_{15}^{18}	$\mathcal{C}_{14}^4 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{19}_{15}	$C_{14}^3 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 128$	C_{15}^{20}	$\mathcal{C}_{14}^{12} \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 384$
\mathcal{C}^{21}_{15}	$C_{14}^{26} \cup \{(0,1,0,1,1,1)\}$	$ \mathcal{O} = 720$	\mathcal{C}^{22}_{15}	$\mathcal{C}_{14}^2 \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2 imes\mathcal{C}_2$
C_{15}^{23}	$C_{14}^{28} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 96$	C_{15}^{24}	$\mathcal{C}_{14}^{39} \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 2304$
\mathcal{C}^{25}_{15}	$C_{14}^2 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 128$	C_{15}^{26}	$\mathcal{C}_{14}^7 \cup \{(0,0,1,1,0,1)\}$	\mathcal{D}_8
\mathcal{C}^{27}_{15}	$C_{14}^9 \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 48$	C_{15}^{28}	$\mathcal{C}_{14}^{19} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 48$
C_{15}^{29}	$C_{14}^2 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 32$	C_{15}^{30}	$\mathcal{C}_{14}^{20} \cup \{(1,0,1,0,0,1)\}$	\mathcal{D}_5
C_{15}^{31}	$C_{14}^{47} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 2688$	C_{15}^{32}	$\mathcal{C}_{14}^2 \cup \{(0,1,0,1,1,0)\}$	\mathcal{D}_4
C_{15}^{33}	$C_{14}^5 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 384$	C_{15}^{34}	$\mathcal{C}_{14}^{15} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 144$
C_{15}^{35}	$C_{14}^{12} \cup \{(1,0,0,1,0,1)\}$	$ \mathcal{O} = 64$	C_{15}^{36}	$\mathcal{C}_{14}^7 \cup \{(0,1,1,0,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$
C_{15}^{37}	$C_{14}^8 \cup \{(1,0,0,1,0,1)\}$	\mathcal{D}_6	C_{15}^{38}	$\mathcal{C}_{14}^{15} \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 48$
C_{15}^{39}	$C_{14}^7 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^{40}_{15}	$\mathcal{C}_{14}^2 \cup \{(0,0,1,1,1,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2 imes\mathcal{C}_2$
\mathcal{C}^{41}_{15}	$C_{14}^{18} \cup \{(1,1,1,1,0,0)\}$	$ \mathcal{O} = 240$	\mathcal{C}^{42}_{15}	$C_{14}^4 \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 48$
C_{15}^{43}	$C_{14}^3 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 192$	C_{15}^{44}	$\mathcal{C}_{14}^6 \cup \{(0,0,0,1,1,1)\}$	$\mathcal{D}_6 \times \mathcal{C}_2$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{16}	$C_{15}^3 \cup \{(1,0,1,1,1,1)\}$	\mathcal{D}_5	\mathcal{C}^2_{16}	$C_{15}^1 \cup \{(1,1,1,0,0,0)\}$	\mathcal{D}_4
\mathcal{C}^3_{16}	$C_{15}^{25} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^4_{16}	$C_{15}^{14} \cup \{(0,1,1,0,0,1)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$
\mathcal{C}^5_{16}	$\mathcal{C}_{15}^{12} \cup \{(0,1,0,1,1,1)\}$	$ \mathcal{O} = 128$	\mathcal{C}^6_{16}	$C_{15}^5 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 768$
\mathcal{C}^7_{16}	$\mathcal{C}_{15}^9 \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 576$	\mathcal{C}^8_{16}	$C_{15}^{17} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 384$
\mathcal{C}^9_{16}	$C_{15}^6 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 192$	\mathcal{C}^{10}_{16}	$C_{15}^{21} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 144$
\mathcal{C}^{11}_{16}	$C_{15}^7 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 32$	C_{16}^{12}	$C_{15}^{22} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 36$
\mathcal{C}^{13}_{16}	$C_{15}^{15} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 192$	C_{16}^{14}	$C_{15}^8 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{15}_{16}	$C_{15}^2 \cup \{(1,0,0,1,1,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	C_{16}^{16}	$C_{15}^{26} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 192$
\mathcal{C}^{17}_{16}	$C_{15}^1 \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	C_{16}^{18}	$C_{15}^6 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 256$
\mathcal{C}^{19}_{16}	$C_{15}^{11} \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 576$	C_{16}^{20}	$C_{15}^{19} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 1536$
\mathcal{C}^{21}_{16}	$C_{15}^{16} \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 11520$	C_{16}^{22}	$C_{15}^{31} \cup \{(1,1,1,1,1,1)\}$	$ \mathcal{O} = 2688$
C_{16}^{23}	$C_{15}^1 \cup \{(0,0,1,1,1,0)\}$	\mathcal{S}_3	C_{16}^{24}	$C_{15}^4 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{25}_{16}	$C_{15}^9 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 384$	C_{16}^{26}	$C_{15}^5 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^{27}_{16}	$C_{15}^{13} \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 720$	C_{16}^{28}	$C_{15}^4 \cup \{(1,0,0,1,1,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2 imes\mathcal{C}_2$
C_{16}^{29}	$C_{15}^2 \cup \{(1,1,0,1,1,1)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$	C_{16}^{30}	$C_{15}^1 \cup \{(1, 1, 0, 0, 0, 1)\}$	$ \mathcal{O} = 64$
C_{16}^{31}	$C_{15}^{17} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 5376$	C_{16}^{32}	$C_{15}^1 \cup \{(1,0,1,1,0,0)\}$	$-2c_1 \times \mathcal{C}_2$
C_{16}^{33}	$C_{15}^4 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 192$	C_{16}^{34}	$C_{15}^9 \cup \{(0,1,1,1,0,1)\}$	$ \mathcal{O} = 48$
C_{16}^{35}	$C_{15}^{16} \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 720$	C_{16}^{36}	$C_{15}^6 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{37}_{16}	$\mathcal{C}_{15}^2 \cup \{(1,1,1,0,0,0)\}$	\mathcal{D}_4	C_{16}^{38}	$C_{15}^{18} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 192$
\mathcal{C}^{39}_{16}	$C_{15}^{31} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 20160$	C_{16}^{40}	$C_{15}^2 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 32$
C_{16}^{41}	$C_{15}^{12} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 192$	C_{16}^{42}	$C_{15}^2 \cup \{(0,1,0,1,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2 imes\mathcal{C}_2$
C_{16}^{43}	$C_{15}^6 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 32$	C_{16}^{44}	$C_{15}^{24} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 36864$
\mathcal{C}^{45}_{16}	$C_{15}^{34} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 576$	C_{16}^{46}	$C_{15}^1 \cup \{(1,0,1,0,1,0)\}$	\mathcal{S}_3
\mathcal{C}^{47}_{16}	$C_{15}^6 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 1024$	C_{16}^{48}	$C_{15}^3 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 60$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
$\overline{\mathcal{C}}_{17}^1$	$C_{16}^7 \cup \{(0,1,1,1,0,1)\}$	$ \mathcal{O} = 576$	$\overline{\mathcal{C}}_{17}^2$	$C_{16}^{14} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 384$
$\overline{\mathcal{C}}_{17}^3$	$C_{16}^{14} \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 720$	$\overline{\mathcal{C}}_{17}^4$	$\mathcal{C}_{16}^{21} \cup \{(0,0,1,1,1,1)\}$	$ \mathcal{O} = 11520$
$\overline{\mathcal{C}}_{17}^{5}$	$C_{16}^{22} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 40320$	\mathcal{C}^6_{17}	$\mathcal{C}_{16}^2 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^7_{17}	$\mathcal{C}_{16}^{8} \cup \{(1,1,1,0,1,1)\}$	$ \mathcal{O} = 96$	\mathcal{C}^8_{17}	$C_{16}^1 \cup \{(1,0,1,1,0,0)\}$	\mathcal{D}_5
\mathcal{C}^9_{17}	$\mathcal{C}_{16}^2 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 64$	\mathcal{C}^{10}_{17}	$\mathcal{C}_{16}^{17} \cup \{(1,1,0,0,0,1)\}$	\mathcal{D}_4
C_{17}^{11}	$C_{16}^1 \cup \{(1,0,1,0,0,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	C_{17}^{12}	$\mathcal{C}_{16}^7 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 2304$
\mathcal{C}^{13}_{17}	$C_{16}^9 \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$	C_{17}^{14}	$\mathcal{C}_{16}^{22} \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 2688$
\mathcal{C}^{15}_{17}	$\mathcal{C}_{16}^2 \cup \{(1,0,1,1,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$	C_{17}^{16}	$\mathcal{C}_{16}^{8} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 2688$
C_{17}^{17}	$C_{16}^{17} \cup \{(1,0,1,1,0,0)\}$	$\mathcal{D}_4 imes\mathcal{C}_2$	C_{17}^{18}	$C_{16}^7 \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 384$
C_{17}^{19}	$\mathcal{C}_{16}^2 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_4	C_{17}^{20}	$\mathcal{C}_{16}^{24} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 48$
C_{17}^{21}	$C_{16}^3 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 48$	C_{17}^{22}	$\mathcal{C}_{16}^3 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 128$
C_{17}^{23}	$C_{16}^7 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 48$	C_{17}^{24}	$C_{16}^1 \cup \{(0, 1, 1, 0, 0, 1)\}$	\mathcal{D}_8
\mathcal{C}^{25}_{17}	$\mathcal{C}_{16}^5 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 769$	C_{17}^{26}	$\mathcal{C}_{16}^6 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 384$
C_{17}^{27}	$C_{16}^{39} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 322560$	C_{17}^{28}	$C_{16}^{13} \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 48$
C_{17}^{29}	$\mathcal{C}_{16}^4 \cup \{(0,0,1,0,1,1)\}$	\mathcal{D}_6	C_{17}^{30}	$C_{16}^{10} \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 288$
C_{17}^{31}	$\mathcal{C}_{16}^1 \cup \{(0,0,1,0,1,1)\}$	\mathcal{S}_3	C_{17}^{32}	$\mathcal{C}_{16}^2 \cup \{(1,0,1,0,1,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$
C_{17}^{33}	$C_{16}^{21} \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 720$	C_{17}^{34}	$\mathcal{C}_{16}^{11} \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 144$
C_{17}^{35}	$\mathcal{C}_{16}^4 \cup \{(0,0,1,1,1,0)\}$	$\mathcal{D}_4 imes\mathcal{C}_2$	C_{17}^{36}	$\mathcal{C}_{16}^{15} \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 32$
C_{17}^{37}	$\mathcal{C}_{16}^9 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 128$	C_{17}^{38}	$\mathcal{C}_{16}^{19} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 9216$
C_{17}^{39}	$C_{16}^{13} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^{40}_{17}	$\mathcal{C}_{16}^3 \cup \{(0,1,1,1,0,0)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
$\overline{\mathcal{C}}^1$	$C_{17}^{14} \cup \{(1,1,1,1,0,0)\}$	$ \mathcal{O} = 10752$	\mathcal{C}^2_{18}	$C_{17}^{21} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 128$
\mathcal{C}^3_{18}	$C_{17}^{22} \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 64$	\mathcal{C}^4_{18}	$C_{17}^{35} \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 192$
\mathcal{C}^5_{18}	$C_{17}^{12} \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 384$	\mathcal{C}^6_{18}	$C_{17}^{13} \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^7_{18}	$C_{17}^{35} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^8_{18}	$\mathcal{C}_{17}^8 \cup \{(0,0,1,0,1,1)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$
\mathcal{C}^9_{18}	$C_{17}^{13} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 144$	\mathcal{C}^{10}_{18}	$\mathcal{C}_{17}^8 \cup \{(0,1,1,0,0,1)\}$	\mathcal{D}_4
C_{18}^{11}	$C_{17}^8 \cup \{(1,0,1,0,0,1)\}$	$\mathcal{C}_2 imes \mathcal{C}_2 imes \mathcal{C}_2$	\mathcal{C}^{12}_{18}	$\mathcal{C}_{17}^{13} \cup \{(1,0,1,1,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
C_{18}^{13}	$C_{17}^9 \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 64$	\mathcal{C}^{14}_{18}	$\mathcal{C}_{17}^6 \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_6
\mathcal{C}^{15}_{18}	$C_{17}^{26} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 6144$	\mathcal{C}^{16}_{18}	$\mathcal{C}_{17}^{15} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{17}_{18}	$C_{17}^7 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 64$	\mathcal{C}^{18}_{18}	$\mathcal{C}_{17}^6 \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{19}_{18}	$C_{17}^7 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 576$	\mathcal{C}^{20}_{18}	$\mathcal{C}_{17}^6 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 384$
\mathcal{C}^{21}_{18}	$C_{17}^{12} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 1536$	C_{18}^{22}	$\mathcal{C}_{17}^{17} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 96$
C_{18}^{23}	$C_{17}^6 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_6	C_{18}^{24}	$\mathcal{C}_{17}^{11} \cup \{(1,1,1,1,1,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2$
\mathcal{C}^{25}_{18}	$C_{17}^{11} \cup \{(1,0,0,1,1,0)\}$	$\mathcal{D}_4 imes \mathcal{C}_2$	\mathcal{C}^{26}_{18}	$\mathcal{C}_{17}^{10} \cup \{(1,0,1,0,1,0)\}$	\mathcal{D}_5
\mathcal{C}^{27}_{18}	$C_{17}^7 \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 384$	C_{18}^{28}	$\mathcal{C}_{17}^{26} \cup \{(1,1,1,1,0,0)\}$	$ \mathcal{O} = 1152$
\mathcal{C}^{29}_{18}	$C_{17}^{17} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 96$	\mathcal{C}^{30}_{18}	$\mathcal{C}_{17}^9 \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_4
C_{18}^{31}	$C_{17}^{16} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 43008$	C_{18}^{32}	$\mathcal{C}_{17}^{10} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 32$
C_{18}^{33}	$\mathcal{C}_{17}^{22} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 2304$			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{19}	$C_{18}^2 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 1152$	\mathcal{C}^2_{19}	$C_{18}^9 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 36$
\mathcal{C}^3_{19}	$C_{18}^7 \cup \{(0,0,1,0,1,1)\}$	\mathcal{S}_3	\mathcal{C}^4_{19}	$C_{18}^4 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^5_{19}	$C_{18}^3 \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^6_{19}	$C_{18}^{15} \cup \{(1,1,1,1,0,0)\}$	$ \mathcal{O} = 9216$
\mathcal{C}^7_{19}	$C_{18}^5 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 144$	\mathcal{C}^8_{19}	$C_{18}^3 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 192$
\mathcal{C}^9_{19}	$C_{18}^{17} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 192$	\mathcal{C}^{10}_{19}	$C_{18}^{14} \cup \{(1,0,1,0,1,0)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{11}_{19}	$C_{18}^4 \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 48$	C_{19}^{12}	$C_{18}^{13} \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 96$
C_{19}^{13}	$C_{18}^{12} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 48$	C_{19}^{14}	$C_{18}^{14} \cup \{(0,1,1,0,1,0)\}$	$\mathcal{D}_6 imes \mathcal{C}_2$
\mathcal{C}^{15}_{19}	$C_{18}^{16} \cup \{(0,0,1,1,1,0)\}$	$ \mathcal{O} = 48$	C_{19}^{16}	$C_{18}^{10} \cup \{(1,0,1,0,0,1)\}$	\mathcal{D}_8
\mathcal{C}^{17}_{19}	$C_{18}^8 \cup \{(0,1,0,0,1,1)\}$	$\mathcal{C}_2 \times \mathcal{C}_2$	C_{19}^{18}	$C_{18}^6 \cup \{(0,1,1,1,0,0)\}$	\mathcal{D}_6
\mathcal{C}^{19}_{19}	$C_{18}^5 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 128$	C_{19}^{20}	$C_{18}^2 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{21}_{19}	$C_{18}^3 \cup \{(0,0,1,0,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	\mathcal{C}^{22}_{19}	$C_{18}^8 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_4
C_{19}^{23}	$C_{18}^{19} \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 2304$	C_{19}^{24}	$C_{18}^2 \cup \{(0,1,0,1,1,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^{25}_{19}	$C_{18}^{19} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 9216$			

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
$\overline{\mathcal{C}}_{20}^1$	$C_{19}^6 \cup \{(0,1,1,1,1,0)\}$	$ \mathcal{O} = 184320$	\mathcal{C}^2_{20}	$C_{19}^{11} \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 128$
\mathcal{C}^3_{20}	$C_{19}^{8} \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 1536$	\mathcal{C}^4_{20}	$\mathcal{C}_{19}^3 \cup \{(0,0,1,1,0,1)\}$	$ \mathcal{O} = 120$
\mathcal{C}^5_{20}	$C_{19}^{15} \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 384$	\mathcal{C}^6_{20}	$C_{19}^3 \cup \{(1,0,0,1,1,0)\}$	\mathcal{D}_4
\mathcal{C}^7_{20}	$C_{19}^{10} \cup \{(0,0,1,1,1,0)\}$	\mathcal{D}_6	\mathcal{C}^8_{20}	$C_{19}^8 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^9_{20}	$C_{19}^2 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 144$	\mathcal{C}^{10}_{20}	$C_{19}^1 \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 23040$
\mathcal{C}^{11}_{20}	$C_{19}^3 \cup \{(1,0,1,1,0,0)\}$	\mathcal{D}_4	\mathcal{C}^{12}_{20}	$\mathcal{C}_{19}^5 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{13}_{20}	$C_{19}^7 \cup \{(1,1,1,0,0,0)\}$	$ \mathcal{O} = 32$	C_{20}^{14}	$C_{19}^{13} \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 96$
\mathcal{C}^{15}_{20}	$C_{19}^9 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 120$	C_{20}^{16}	$C_{19}^{10} \cup \{(1,0,0,1,1,0)\}$	$ \mathcal{O} = 32$
\mathcal{C}^{17}_{20}	$C_{19}^9 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 192$	\mathcal{C}^{18}_{20}	$\mathcal{C}_{19}^5 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 256$
\mathcal{C}^{19}_{20}	$C_{19}^9 \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 3072$	C_{20}^{20}	$C_{19}^3 \cup \{(1,0,1,1,1,1)\}$	\mathcal{D}_6
\mathcal{C}^{21}_{20}	$C_{19}^2 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 96$	\mathcal{C}^{22}_{20}	$\mathcal{C}_{19}^{23} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 36864$
C_{20}^{23}	$C_{19}^1 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 96$	C_{20}^{24}	$C_{19}^2 \cup \{(1,0,1,1,0,0)\}$	$\mathcal{C}_2 imes\mathcal{C}_2 imes\mathcal{C}_2$

\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{21}	$C_{20}^4 \cup \{(1,0,1,1,0,0)\}$	\mathcal{D}_5	\mathcal{C}^2_{21}	$\mathcal{C}_{20}^3 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 192$
\mathcal{C}^3_{21}	$C_{20}^6 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^4_{21}	$\mathcal{C}_{20}^6 \cup \{(1,0,1,1,0,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^5_{21}	$C_{20}^7 \cup \{(1,0,1,1,0,0)\}$	$\mathcal{D}_6 \times \mathcal{C}_2$	\mathcal{C}^6_{21}	$\mathcal{C}_{20}^6 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^7_{21}	$C_{20}^{10} \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 1920$	\mathcal{C}^8_{21}	$\mathcal{C}_{20}^2 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 32$
\mathcal{C}^9_{21}	$C_{20}^{17} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 3072$	\mathcal{C}^{10}_{21}	$C_{20}^7 \cup \{(1, 1, 0, 0, 0, 1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^{11}_{21}	$C_{20}^7 \cup \{(1,0,0,1,1,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	\mathcal{C}^{12}_{21}	$\mathcal{C}_{20}^{15} \cup \{(1,1,0,1,1,1)\}$	$ \mathcal{O} = 1920$
C_{21}^{13}	$C_{20}^{15} \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 720$	\mathcal{C}^{14}_{21}	$\mathcal{C}_{20}^{12} \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 1008$
\mathcal{C}^{15}_{21}	$C_{20}^2 \cup \{(0,0,1,0,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^{16}_{21}	$C_{20}^3 \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 384$
\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{22}	$C_{21}^6 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 384$	\mathcal{C}^2_{22}	$C_{21}^3 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 192$
\mathcal{C}^3_{22}	$C_{21}^2 \cup \{(0,1,0,1,1,0)\}$	$ \mathcal{O} = 48$	\mathcal{C}^4_{22}	$C_{21}^4 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 144$
\mathcal{C}^5_{22}	$C_{21}^9 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 768$	\mathcal{C}^6_{22}	$C_{21}^9 \cup \{(1,1,1,1,1,0)\}$	$ \mathcal{O} = 6144$
\mathcal{C}^7_{22}	$C_{21}^1 \cup \{(1,0,1,1,1,1)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$	\mathcal{C}^8_{22}	$C_{21}^1 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 120$
\mathcal{C}^9_{22}	$C_{21}^{14} \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 2688$	\mathcal{C}^{10}_{22}	$C_{21}^2 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 3840$
\mathcal{C}^{11}_{22}	$C_{21}^{12} \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 11520$	$\mathcal{C}_{22}^{\overline{12}}$	$C_{21}^1 \cup \{(1,0,0,1,1,0)\}$	$\mathcal{D}_4 \times \mathcal{C}_2$
\mathcal{C}^{13}_{22}	$C_{21}^5 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 72$	\mathcal{C}^{14}_{22}	$\mathcal{C}_{21}^2 \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 64$
\mathcal{C}^{15}_{22}	$C_{21}^6 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 1008$			
\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{23}	$C_{22}^6 \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 21504$	\mathcal{C}^2_{23}	$C_{22}^3 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 768$
\mathcal{C}^3_{23}	$C_{22}^2 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 32$	\mathcal{C}^4_{23}	$C_{22}^2 \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 48$
\mathcal{C}^5_{23}	$C_{22}^5 \cup \{(0,1,1,1,0,0)\}$	$ \mathcal{O} = 1152$	\mathcal{C}^6_{23}	$C_{22}^1 \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 384$
\mathcal{C}^{7}_{23}	$C_{22}^4 \cup \{(1,0,1,1,1,1)\}$	$ \mathcal{O} = 48$	\mathcal{C}^8_{23}	$C_{22}^1 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 144$
\mathcal{C}^9_{23}	$C_{22}^3 \cup \{(0,1,1,1,1,1)\}$	$ \mathcal{O} = 336$			
\mathcal{K}	CORRESPONDS TO	STABILIZER	\mathcal{K}	CORRESPONDS TO	STABILIZER

 $|\mathcal{O}| = 516096$

 $|\mathcal{O}| = 512$

 $|\mathcal{O}| = 6144$

 $|\mathcal{O}| = 96$

 $C_{23}^7 \cup \{(1,1,0,1,1,1)\}$

 $C_{23}^4 \cup \{(0, 1, 0, 1, 0, 1)\}\$ $C_{23}^3 \cup \{(1, 0, 1, 1, 1, 1)\}\$

 $C_{23}^1 \cup \{(0,1,1,0,0,1)\}$

 $|\mathcal{O}| = 1152$

 $|\mathcal{O}| = 192$

 $|\mathcal{O}| = 72$

 $|\mathcal{O}| = 2688$

 $C_{23}^1 \cup \{(0,0,1,1,0,1)\}$

 $C_{23}^2 \cup \{(1,0,1,1,1,1)\}$ $C_{23}^2 \cup \{(1,0,1,0,0,1)\}$

 $C_{23}^{\overline{3}} \cup \{(0,1,0,0,1,1)\}$

 $\mathcal{C}^2_{24} \ \mathcal{C}^4_{24} \ \mathcal{C}^6_{24} \ \mathcal{C}^8_{24}$

\mathcal{K}	CORRESPONDS TO	STABILIZER
\mathcal{C}^1_{25}	$C_{24}^4 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 720$
\mathcal{C}^2_{25}	$C_{24}^2 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 144$
\mathcal{C}^3_{25}	$C_{24}^3 \cup \{(0,1,0,0,1,1)\}$	$ \mathcal{O} = 1152$
\mathcal{C}^4_{25}	$C_{24}^3 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 768$
\mathcal{C}^5_{25}	$C_{24}^1 \cup \{(0,1,1,0,0,1)\}$	$ \mathcal{O} = 64512$
\mathcal{C}^1_{26}	$C_{25}^2 \cup \{(1,1,1,1,0,1)\}$	$ \mathcal{O} = 768$
\mathcal{C}^2_{26}	$C_{25}^1 \cup \{(1,0,1,1,0,0)\}$	$ \mathcal{O} = 720$
\mathcal{C}^3_{26}	$C_{25}^1 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 11520$
\mathcal{C}^4_{26}	$C_{25}^4 \cup \{(0,0,1,1,0,1)\}$	$ \mathcal{O} = 18432$
\mathcal{C}^1_{27}	$C_{26}^1 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 1920$
\mathcal{C}^2_{27}	$C_{26}^1 \cup \{(1,1,0,1,0,0)\}$	$ \mathcal{O} = 9216$
\mathcal{C}^1_{28}	$C_{27}^2 \cup \{(0,1,1,0,1,0)\}$	$ \mathcal{O} = 258048$
\mathcal{C}^2_{28}	$C_{27}^1 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 9216$
\mathcal{C}^1_{29}	$C_{28}^1 \cup \{(1,0,1,0,0,1)\}$	$ \mathcal{O} = 64512$
\mathcal{C}^1_{30}	$C_{29}^1 \cup \{(0,1,0,1,0,1)\}$	$ \mathcal{O} = 645120$
\mathcal{C}^1_{31}	$C_{30}^1 \cup \{(1,1,0,0,0,1)\}$	$ \mathcal{O} = 9999360$
$\overline{\mathcal{C}}_{32}^1$	$C_{31}^1 \cup \{(0,0,1,1,0,1)\}$	$ \mathcal{O} = 319979520$

All the complete caps of sizes 13, 17, 18 and 20 have projective frame. The 32- complete cap has no projective frame.

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